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Two-Mode Waveguide for Equal Mode Velocities: Correction

N. G. ALEXOPOULOS AND M. E. ARMSTRONG

***Abstract*—The T-septum waveguide was analyzed by Elliott using the orthonormal block method. The numerical results did not compare favorably with experimental measurements and it was suggested that the disparity was related primarily to the assumption of zero-thickness membranes for the septum. Later, Silvester analyzed the T-septum waveguide using a finite-element method and found very good agreement with the measured points, yet the septum thickness was again assumed to be infinitesimal. This letter is being written to dispel the implication that the orthonormal block method of analysis of the T-septum waveguide suffers for lack of accuracy. The universal curves as shown by Elliott will be presented here in corrected form along with experimental results further corroborating both Elliott's and Silvester's work.**

Elliott's analysis [1] of the T-septum waveguide using the orthonormal block method has been corroborated by recent numerical calculations utilizing his theoretical formulation and by additional experimental measurements. It has been determined that errors existed in the original computer program used for the determination, numerically, of cutoff wavenumbers from the difference-mode Rayleigh–Ritz approximation.

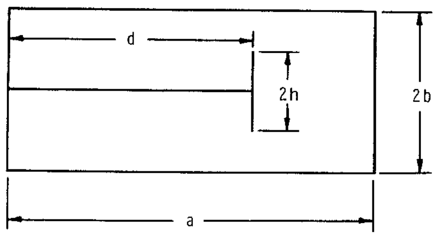

Fig. 1. T-septum waveguide geometry.

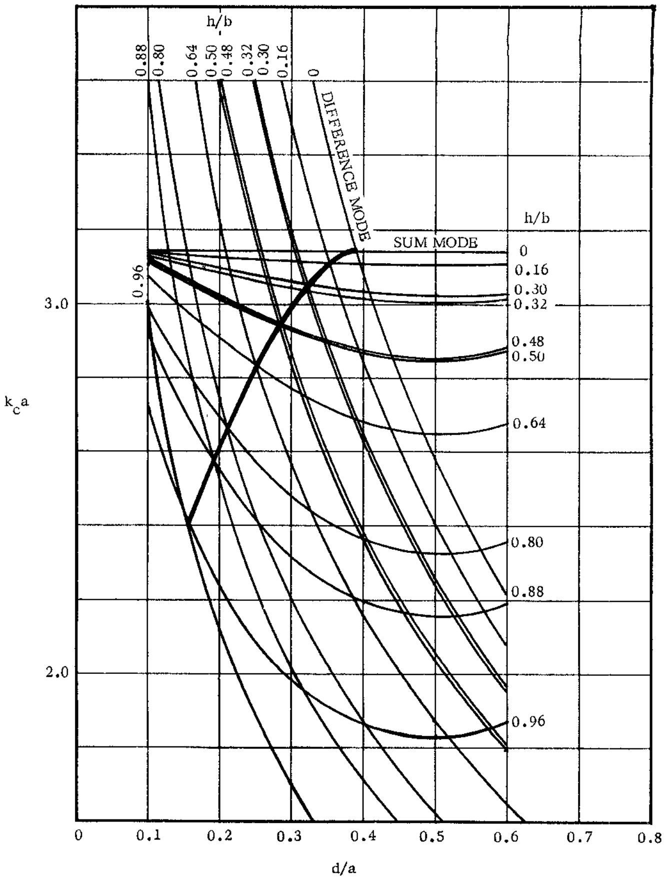

Fig. 2. Cutoff wavenumbers for sum and difference modes.

The T-septum waveguide geometry is indicated in Fig. 1, and the corrected cutoff wavenumbers for the sum and difference modes are shown in Fig. 2 for a range of septum dimensions. Fig. 3 shows the calculated guide wavelengths for the cases $h/b=0.3$, 0.5 and 0.8, along with experimentally determined guide wavelengths for these same T-septum aspect ratios. The heavy solid curves (theoretical) and the heavy dashed curve (experimental) in Figs. 2 and 3 give d as a function of h such that, over the frequency range for which these two modes propagate, their phase velocities will be equal. It is noted that there is satisfactory correspondence between experimental and theoretical characteristics for moderate septum insertion depths and that the results deviate markedly for large insertion depths, possibly because of the considerable difference between theoretical and experimental T-septum models. However, in the region of interest, namely for those values of insertion depth for which the phase velocities of the two modes are equal, there is excellent agreement, indicating, therefore, the validity of the theoretical model for predicting the physical model characteristics.

For purposes of comparison the current theoretical and experimental data are presented in Fig. 4 ($h/b=0.3$) along with the theoretical characteristics determined earlier by Silvester ($h/b=0.3$) [2]

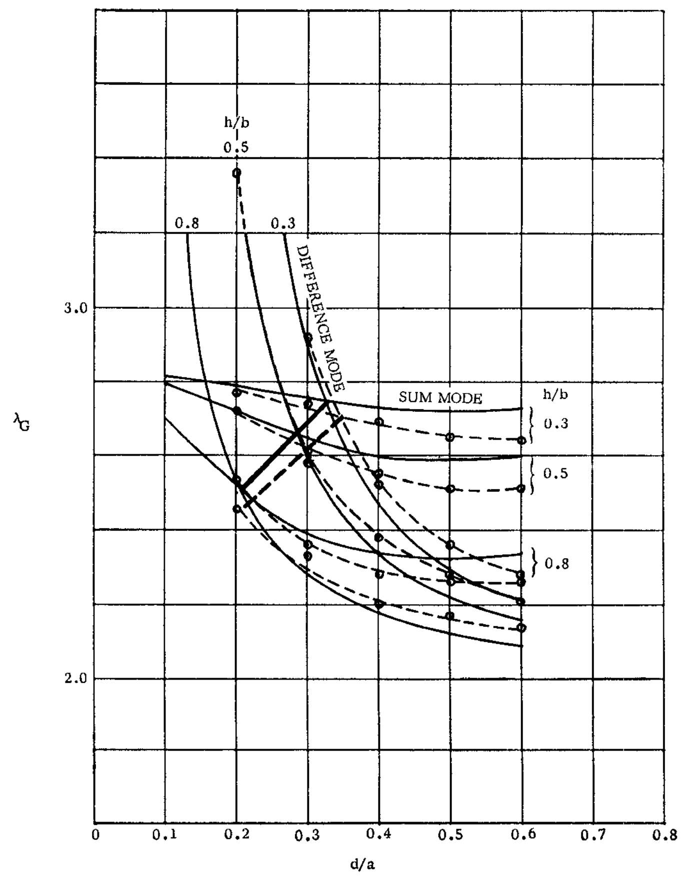

Fig. 3. Guide wavelengths of 6 GHz for sum and difference modes. (————) theoretical. (⊙------⊙) experimental

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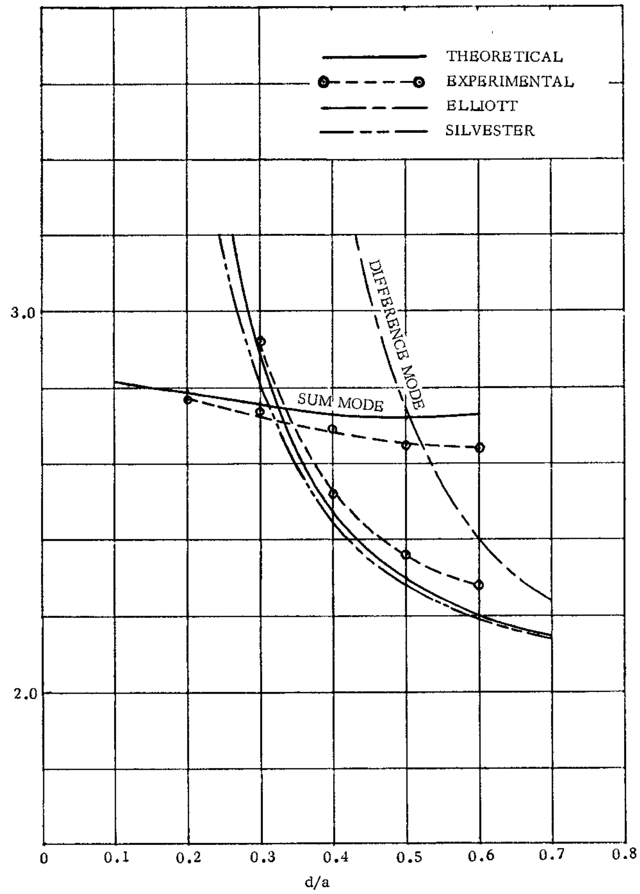

Fig. 4. Comparison of results.

and Elliott ($h/b=0.32$). The theoretical-sum mode characteristic nearly matches for the three cases and is not shown.

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On the Solution of the Circularly Cylindrical Coordinate Wave Equation in Homogeneous Isotropic Regions Containing the Coordinate Axis

D. M. BOLLE

***Abstract*—In a number of well-known texts, misleading statements are made concerning the reason for eliminating the Neumann or Bessel function of the second kind from the solution of the wave equations in circular cylindrical coordinates for a homogeneous region containing the coordinate axis. This letter discusses the conditions that are required to arrive at a unique solution.**

In several texts generally consulted by students in the field of electromagnetic theory [1]–[5], erroneous and misleading statements are made concerning the reason for excluding the Bessel functions of the second kind from the solution of the wave equation in circular cylindrical coordinates in isotropic homogeneous regions containing the axis. The condition used is that the field quantities, in these cases E_Z or H_Z, must be bounded. Two objections to such a condition must be made. First, the statement is misleading in that students infer too readily that field components must always be bounded in such situations. Such is certainly not the case, e.g., in geometries where the boundaries exhibit sharp edges. Secondly, such a statement or condition does not rest on a demonstrable physical principle.

The finite energy condition, i.e., square integrability, can be invoked at this point and may be stated in the form

$$\int_0^{\delta V} |\psi|^2 dv \leq M.$$

Now, whenever higher order modes are considered where

$$\psi \propto [J_n(x), N_n(x)] \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} n\phi, \qquad n \neq 0$$

and

$$N_n(x) \sim x^{-n} (x \to 0), \qquad n = 1, 2, 3, \cdots$$

then the finite energy condition suffices to prohibit the use of the Neumann functions.

However, for regions with rotational symmetry where the lowest order admissable solution is

$$\psi \propto \lfloor J_0(x), N_0(x) \rfloor$$

and

$$N_0(x) \sim \log x (x \to 0)$$

there results

$$\int_0^{\delta x} x (\log x)^2 dx \leq M_0.$$

Hence, in this case, the finite energy condition is not sufficient.

This question is resolved only when, instead of restricting our attention to the axial field components, all the field components are examined. For the lowest order mode the following result is then obtained:

$$(H_r, E_r) \propto N_1(x) \sim x^{-1} (x \to 0)$$

and it is observed that, unless sources are present on the axis, such a solution is not acceptable.

Thus the crucial condition in rejection Bessel functions of the second kind is not boundedness nor finite energy, but that the region must be source free!

When the fractional-order Bessel functions arise, as is the case in reentrant sectorial cylindrical guides, then again the finite energy condition on the axial field components is not sufficient, but the radial components must be examined. The order of the singular behavior is such that Neumann solutions would indicate line sources on the edge of the reentrant sector. Thus, unless appropriate line sources are specified, such solutions must be eliminated.

A further related point should be considered. In problems involving cylindrical geometries, in particular the circular cylinder or coaxial circular cylinder, the solution of the wave equations yields trigonometric functions in the angular coordinate. If radial planes restrict the range of the angular coordinate, then the separation constant multiplying the angular coordinate in the argument of the trigonometric functions is determined explicitly by the boundary condition on the radial planes. However, if no such subdivision occurs, then, too often, the single-valuedness condition on the field quantities is used to obtain the undoubtedly correct result that the separation constant must be an integer. The single valuedness of the solution can only be used if the angular variable is allowed to range over $-\infty < \phi < \infty$, say, and such need not be the case since restricting the range of ϕ to $-\pi \leq \phi < \pi$, or $\phi_0 \leq \phi < \phi_0 + 2\pi$, does not detract from the generality of the problem solution. In that event, single valuedness cannot be used, and the integral value of the separation constant must be arrived at through the condition that the field must be everywhere continuous. If any value other than an integral value is taken, a discontinuity in the field will occur at $\phi = \phi_0$, which would indicate the presence of a radial sheet of sources. It then follows that, if no

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